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## 13. Abstract (Maximum 200 words).

The Helmholtz-Poincare Wave Equation (H-PWE) arises in many areas of classical wave scattering theory. In particular it can be found for the cases of acoustical scattering from submerged bounded objects and electromagnetic scattering from objects. The extended boundary integral equations (EBIE) method is derived from considering both the exterior and interior solutions of the H-PWE's. This coupled set of expressions has the advantage of not only offering a prescription for obtaining a solution for the exterior scattering problem, but it also obviates the problem of irregular values corresponding to fictitious interior eigenvalues. Once the coupled equations are derived, they can be obtained in matrix form by expanding all relevant terms in partial wave expansions, including a biorthogonal expansion of the Green function. However, some freedom of choice in the choice of the surface expansion is available since the unknown surface quantities may be expanded in a variety of ways so long as closure is obtained. Out of many possible choices, we develop an optimal method to obtain such expansions which is based on the optimum eigenfunctions related to the surface of the object. In effect, we convert part of the problem (that associated with the Fredholm's integral equation of the first kind) and eigenvalue problem of a related Hermitian operator. The methodology will be explained in detail and examples will be presented.

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Classical Scattering theory of waves from the view point of an eigenvalue problem  
and application to target identification

C. Bottcher and M. R. Strayer

Oak Ridge National Lab, Oak Ridge, TN 37831

and

M. F. Werby

Naval Research Laboratory  
Theoretical Acoustics Branch,  
Stennis Space Center, MS 39529

### ABSTRACT

The Helmholtz-Poincaré Wave Equation (H-PWE) arises in many areas of classical wave scattering theory. In particular it can be found for the cases of acoustical scattering from submerged bounded objects and electromagnetic scattering from objects. The extended boundary integral equations (EBIE) method is derived from considering both the exterior and interior solutions of the H-PWE's. This coupled set of expressions has the advantage of not only offering a prescription for obtaining a solution for the exterior scattering problem, but it also obviates the problem of irregular values corresponding to fictitious interior eigenvalues. Once the coupled equations are derived, they can be obtained in matrix form by expanding all relevant terms in partial wave expansions, including a biorthogonal expansion of the Green function. However some freedom of choice in the choice of the surface expansion is available since the unknown surface quantities may be expanded in a variety of ways so long as closure is obtained. Out of many possible choices, we develop an optimal method to obtain such expansions which is based on the optimum eigenfunctions related to the surface of the object. In effect, we convert part of the problem (that associated with the Fredholms integral equation of the first kind) an eigenvalue problem of a related Hermitian operator. The methodology will be explained in detail and examples will be presented.

### I. INTRODUCTION

The extended boundary condition (EBC), null field, or T-matrix method of Waterman (1969)<sup>1-3</sup> has proven to be a powerful numerical approach for computing scattering from a wide variety of elongated, axisymmetric, impenetrable and elastic objects. A key point in developing the Waterman matrix approach is the inclusion of an integral solution representation for regions both interior and exterior to the object. In the standard ERC formulation, the unknown surface displacements and stresses are expanded on a known (but in principle arbitrary) basis set, with unknown expansion coefficients. Completeness or closure of these basis functions on the surface is generally either known *a priori* or established, with the additional requirement that the number of expansion terms match the surface to the number of incident partial waves. This requirement, however, can in many cases entail more partial waves than strictly necessary for numerical convergence of the incident field, which leads to small incidence higher-order wavefield components. Precisely these components, in turn, render the resulting Q matrix (defined below) potentially ill-conditioned or at the least computationally expensive, using standard matrix techniques. In this paper, we develop a novel alternative for representing the surface terms in the acoustic and elastic case in both stable and efficient form. For an acoustic target, this procedure avoids the ill-conditioned problem by transforming the interior problem to a simpler eigenvalue problem, which produces eigenstates that span the space of surface terms (surface displacement, traction, etc.) on the surface of the object. The eigenstates, with known (derivable) coefficients, can then be used in directly solving the exterior problem, thereby yielding a numerically stable and convergent solution. In the following, a complete theoretical derivation of the eigenexpansion technique is presented. Numerical examples are presented to complete the work.

## II. THE EXTENDED BOUNDARY CONDITION METHOD IN REVIEW

The Helmholtz-Poincaré integral representation for a field interior to a bounded object can be expanded as follows:

$$U(r) = U_i(r) + \int_S [U_+(r') \partial G(r, r') / \partial n - G(r, r') \partial U_+(r') / \partial n] dS \quad (1)$$

where  $r$  is chosen to be at an exterior point to the object; i.e.,  $r$  is a member of  $D$ , where  $D$  is the set of all points exterior to the object,  $G$  is an outgoing Green's function,  $D'$  is the set of points in the interior of the bounded object, and  $S$  is the set of all points on the object surface. The surface of the object is assumed to be piecewise continuous. It is redolent of Gauss' law that when  $r$  is in the interior the total wavefield  $U(r)$  is zero (extinguished or nulled), hence the terms "extinction theorem" or "null-field condition."

$$0 = U_i(r) + \int_S [U_+(r') \partial G(r, r') / \partial n - G(r, r') \partial U_+(r') / \partial n] dS \quad (2)$$

where  $r'$  is an interior point. Let us assume that  $\partial U_+(r') / \partial n = 0$ , so that we obtain the expressions

$$U(r) = U_i(r) + \int_S [U_+(r') \partial G(r, r') / \partial n] dS \quad (3)$$

$$0 = U_i(r) + \int_S [U_+(r') \partial G(r, r') / \partial n] dS. \quad (4)$$

To solve these expressions, it is necessary to represent  $U_i(r)$ ,  $U_+(r')$  and  $G(r, r')$  in some convenient series expansion, which upon truncation would lead to matrix equations that can then be solved using digital computers. The Green's function  $G$  is a normal operator, and thus can be represented in the biorthogonal series

$$G(r, r') = ik \sum \text{Re} \varphi_i(r_<) \varphi_i(r_>) \quad (5)$$

where  $r_<$  and  $r_>$  are the greater and lesser of the two points  $r$  and  $r'$  relative to the origin of the object, respectively. In a manner similar to that of the Hilbert-Schmidt theorem for symmetric kernels,<sup>7</sup> it can be shown that

$$U_i(r) = \sum a_n \text{Re} \varphi_n(r). \quad (6)$$

For incident plane waves, the  $a$ 's are known. We now have from (4) the relation

$$\sum_n a_n \text{Re} \varphi_n(r) = ik \sum_n \int \text{Re} \varphi_n(r) U_+(r') \partial \varphi_i(r') / \partial n dS \quad (7)$$

and

$$a_n = ik \int U_+(r') \partial \varphi_i(r') / \partial n dS. \quad (8)$$

We now wish to represent (8) in matrix form. This can be achieved by writing  $U_+(r')$  in some complete set of known functions<sup>8-11</sup> so that

$$U_+(r) = \sum_n b_n \text{Re} \varphi_n(r) \quad (9)$$

where  $b_n$  is unknown. Expression (8) now becomes

$$a_n = \sum_m b_m \int \text{Re} \varphi_m(r') \partial \text{Re} \varphi_n(r') / \partial n dS = \sum_m b_m Q_{mn}. \quad (10)$$

To obtain the most efficient expansions let us premultiply equation (11) by the adjoint of  $Q$ , namely  $Q^\dagger$ , where the latter quantity is the complex transpose of  $Q$ .

$$Q^\dagger a = i \quad Q^\dagger Q b = i \quad H b \quad (11)$$

where the matrix  $H$  can easily be shown to be self-adjoint or Hermitian. The eigenfunctions can be obtained as follows:

$$HB_i = \lambda_i B_i. \quad (12)$$

Here, the adjoint of  $B$  is  $B^\dagger$ , so that  $B_i B_j^\dagger = \delta_{ij}$ , where  $\delta_{ij}$  is the Kronecker delta function. We also have the ordering  $\lambda_1 < \lambda_2 < \lambda_3 \dots$ , where the dimension of  $H$  is any desired order and relates to the number of surface quantities required in expanding  $U_+$ . We now use the properties (12) and (13) to eliminate the quantities  $\{b_i\}$ . One can show that the  $B_i$ 's are an alternate representation of the  $\partial j_i(r')/\partial n$ 's, with the desired property that they are an orthogonal representation. Thus, we have

$$b = \sum \alpha_i B_i \quad (13)$$

so that expressions (15) and (11) along with (12) and (13) lead to the following useful equation

$$Q^\dagger a = i \sum H \alpha_i B_i = i \sum \lambda_i B_i \quad (14)$$

Thus  $\alpha_i = -i B_i Q' A / \lambda_i$  and  $b = -i \sum B_i Q' A / \lambda_i$ .

We also expand the exterior problem as follows:

$$f_i = ik \int U(r')_+ \text{Re} \partial \varphi_i(r) / \partial n dS \quad (15)$$

$$f_i = ik \sum_j b_j \int \text{Re} \varphi_j(r') \text{Re} \partial \varphi_i(r) / \partial n dS = ik \sum_j b_j Q_{ij}. \quad (16)$$

The final expression for the scattered field in terms of the incident wavefield using Eq. 13 and 16 is thus

$$f = -\sum \text{Re} Q B_i Q' A / \lambda_i B_i. \quad (17)$$

Although the above expression has proven computationally efficient, it is also possible and sometimes necessary to obtain an alternate T-matrix representation. This may be done by means of the following derived relation:

$$T = -\text{Re} Q B I / \varphi B^\dagger Q^\dagger \quad (18)$$

where  $B B^\dagger = B^\dagger B = I$ .

Finally, we show here some of the numerical eigenvalues, computed by implementation of Eq. (21), for the case of a rigid spheroid of  $KL/2 = 10$  and aspect ratio of 5. We show the first three and last two in a twenty term expansion.

$$\lambda_1 = 0.0664; \lambda_2 = 0.0812; \lambda_{19} = 0.318; \lambda_{20} = 0.324. \quad (19)$$

There exists a difference a factor of 5 between the eigenvalues of the first and last orders; for more realistic cases (of higher aspect ratio) differences of 3 orders of magnitude or more are readily possible. Let us now state the advantages of this method of this method over the conventional T-matrix approach which uses matrix inversion. The eigenexpansion method reduces the problem to one of finding eigenvalues for a Hermitian matrix, which is computationally straightforward procedure. For our calculations, we employed the QR algorithm above. The results presented here always maintained numerical stability and were carried out on a computer of 15 place precision. It is not possible to carry out such calculations using the coupled higher order T-matrix method<sup>6</sup> without resorting to 32 place accuracy. The conventional T-matrix approach fails to converge for the problems presented here. The reason for the better precision range using the eigenexpansion method is that the method only breaks down when adjacent eigenvalues are closer in value than a word length while numerical inversion is considerably less stable.

### III. APPLICATION OF METHOD: NUMERICAL RESULTS

By aspect ratio we mean the ratio of length to width (a measure of object elongation; the wavenumber  $k = 2/\lambda$ , where  $L$  is the wavelength of the incident wave in the fluid, and  $\lambda$  is the length of the object. Differential scattering cross-sections or bistatic angular distributions prove useful in classic acoustics and electromagnetic theory in the determination of shape. In nuclear physics, bistatic distributions are typically employed in determining the angular momentum and parity of a nucleus. Monostatic angular distributions, also examined below, are useful in determining object shape factors such as symmetry properties, and aspect ratios. Monostatic angular distributions correspond to backscattered measurements, in which the sensors are effectively rotated about the scatterer (or alternately the object is rotated for a fixed sensor configuration). Backscattered echoes, correspond to fixed the source-receiver configurations in which frequency varies monotonically over small increments (of  $kL/2$ ). We examine these quantities which are often referred to as form functions in acoustics, and correspond to excitation functions in quantum scattering.<sup>10</sup> We list the examples below to be examined:

(a) Scattering from a rigid spheroid with an aspect ratio of 15:1, and a  $kL/2 = 60$

(b) Scattering from a soft spheroid with an aspect ratio of 15:1, and a  $kL/2 = 60$

Case A. In the following, we examine bistatic and monostatic angular distributions, for a rigid spheroid with an aspect ratio of 15 and a  $kL/2$  of 60. Fig. 1 illustrates the case of scattering along the axis of symmetry (where in all that follows, detections are conducted in the plane of the object). Note that we plot the angular distribution over 360 degrees. In all cases, the object is aligned horizontal to the page. A detection at zero degrees will be considered in the backward direction, while a detection at 180 degrees is here taken in the forward direction, the converse of standard convention. Notice that the diffracted wave, which is more pronounced at higher frequencies, tends towards the forward direction. In the optical limit (as wavelength approaches zero), the entire diffracted field would occur in the forward direction, with only specular reflection.

Fig. 2 illustrates the case of scattering at 30 degrees relative to the axis of symmetry. Here the results are both more complex and interesting. Note that there are pronounced response maxima at 150 and 210 degrees, corresponding respectively to a reflected and diffracted wave. This reflected response is typically observed only in the case of very elongated objects, which approach a flat plate, and corresponds to the flat plate limit [incident angle = reflected angle] in the limit of a very long target.

Fig. 3 shows the same features as Fig. 2, for scattering at 60 degrees relative to the axis of symmetry (reflected wave = 120; diffracted wave = 240). Fig. 4 corresponds to scattering at 90 relative to the symmetry axis ("broadside"). Here we observe significant backscattering, due to the large geometrical cross-section, and a very peaked forward scattering response. Note that as scattering angles change from end-on to broadside, the forward scattering increases significantly in value, from 6 to 28 dB, and that the forward response tends to progressively narrow. (We have carried these calculations to  $kL/2$  values of 170 broadside, in which the forward-scattered response is on the order of 1 degree in width. The magnitude differential is 23 dB, between the forward and backscattered fields, irrespective of the target dimensions.

Fig. 5 illustrates a monostatic angular distribution for the same case as above (rigid spheroid). The minima in this case correspond to an interference phenomena, associated with waves reflected at different locations on the surface. We have shown previously<sup>13</sup> that the number of nulls for elongated objects is a function of  $kL/2$ , as follows:

$$N = \text{Int}(kL/2)/\pi - 1). \quad (20)$$

We have also shown that the nulls occur at the following angles:

$$\theta_n = \text{Sin}^{-1}[\pi(n+1/2)/2(kL/2)]. \quad (21)$$

It is clear that in the present case (of  $kL/2 = 60$ ), the number of nulls is quite large ( $N = 148$ ). As one approaches the optical limit, the response minima become progressively less pronounced, and the net response is representative

of an outline of the scattering target. Thus, at suitably high frequencies, it is possible to obtain an indication of both the shape (symmetry and aspect ratio) of an object, by examining the monostatic angular distribution. The above conclusion also proves true both for different object shapes, and boundary conditions.

#### IV. CONCLUSIONS

This paper constitutes a first step in more fully outlining and applying a recently developed eigenexpansion and similarity-type transformation method for studying scattering from spheroidal solids and shells of high aspect ratio. The methods have been rigorously developed to assess and extend the validity range of the EBC/T-matrix method for full wave-theoretic simulation of scattering of plane acoustic or elastic waves by a submerged or embedded single or multilayered target of arbitrary elasto-acoustic composition. The numerical results presented here agree fully with those obtained both by classical normal mode and T-matrix solutions, with the added advantage that less absolute numerical precision is required for the T-matrix method. This, and the fact that the preceding methods are not limited to any specific basis functions, suggest that in addition to extending the range of validity of the EBC method to axisymmetric scatterers of higher aspect ratio, the eigen-expansion and transform approaches also provide improved numerical stability and accuracy at the optical limit in the expansion of the scattered wavefield in partial wave eigenfunctions. Special consideration has been given to interpreting numerical simulations of the excitation and diffraction effects of both simple and circumferential surface waves, as observable in both bistatic angular and spectral frequency resonance distributions. These, as well as generalized Snell's law behaviours, are illustrated. Although for simplicity the present treatment has been restricted to plane wave insonification of single scatterers, the above can also be applied to more realistic cases. In view of these results, this method may also prove useful for improving and/or extending the range of validity for other types of scattering formulations. A promising further development is a hybrid computation of scattering, involving a T-matrix / eigen-expansion method. Work is continuing in further extending and applying these methods, and in developing new techniques to handle cases of both higher frequency, aspect ratio, and target complexity.

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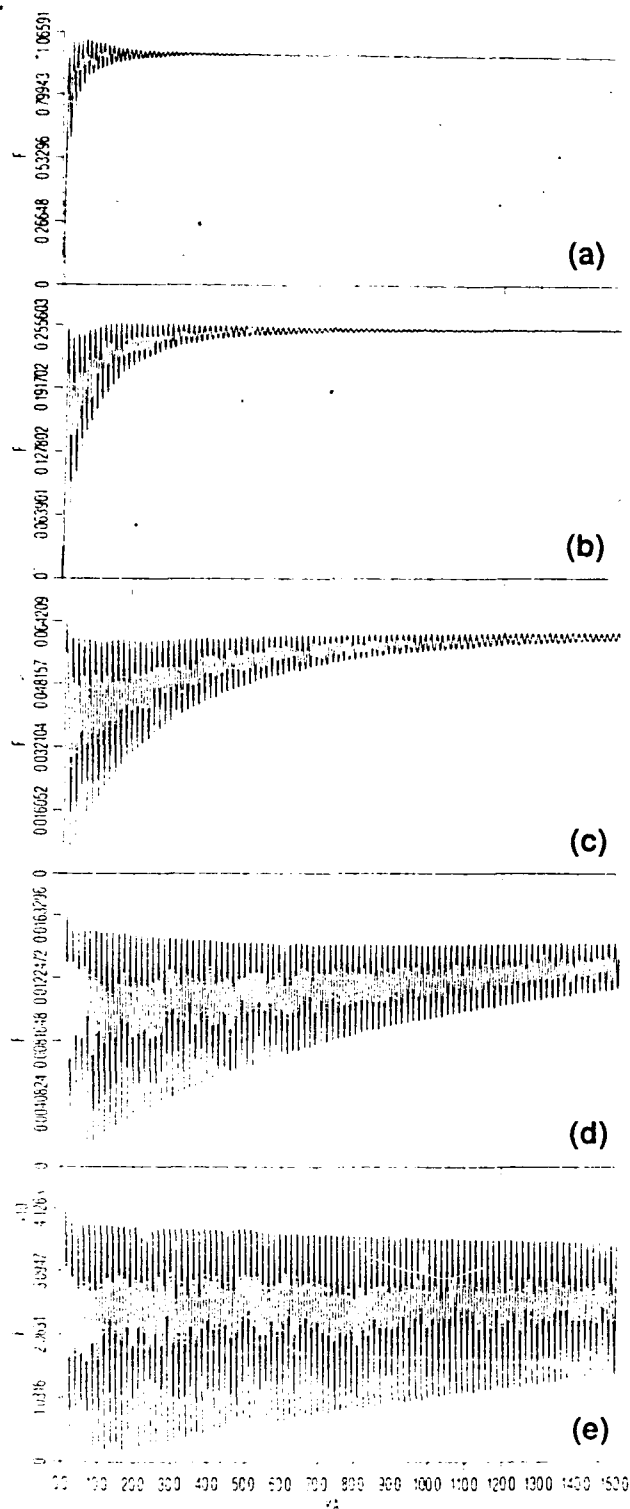


Fig. 1. Backscatter for  $kL/2=0$  to 150 for (a) sphere and spheroid for aspect ratios of (b) 2, (c) 4, (d) 8, and (e) 16.

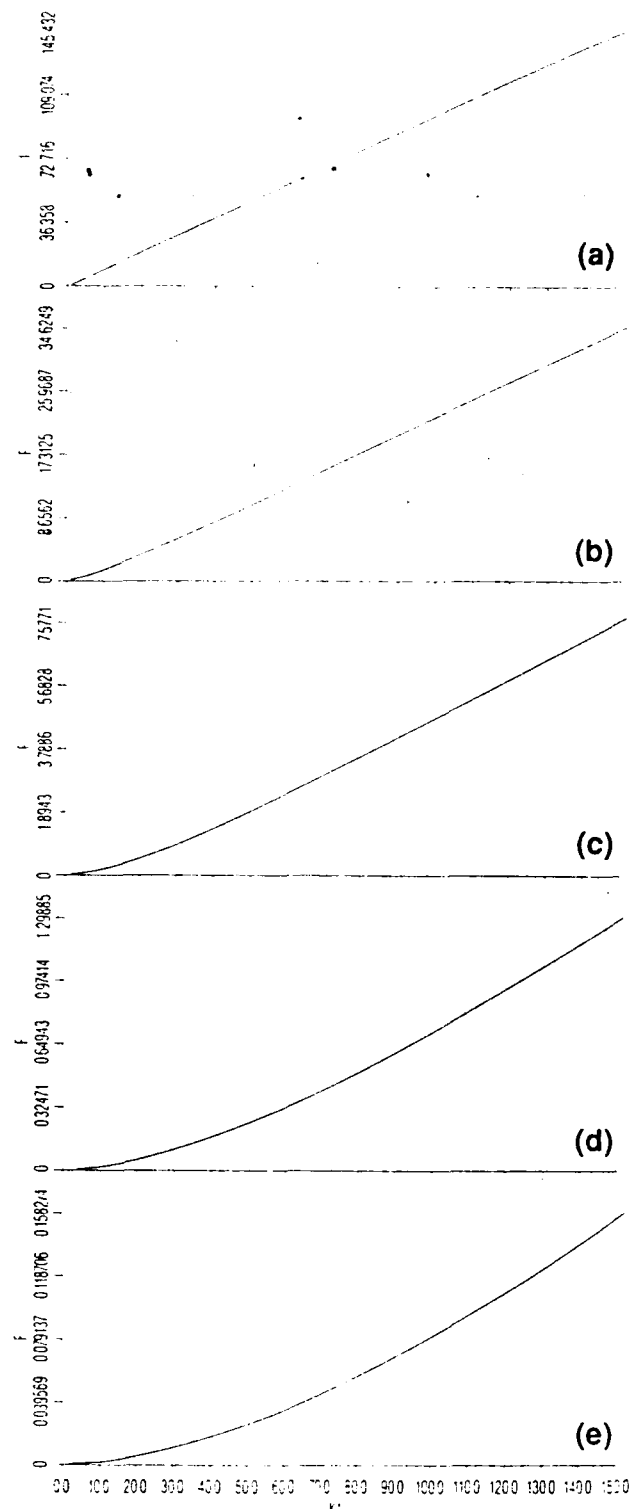


Fig. 2. Forward scatter for  $kL/2=0$  to 150 for (a) sphere and spheroid with aspect ratios of (b) 2, (c) 4, (d) 8, and (e) 16.

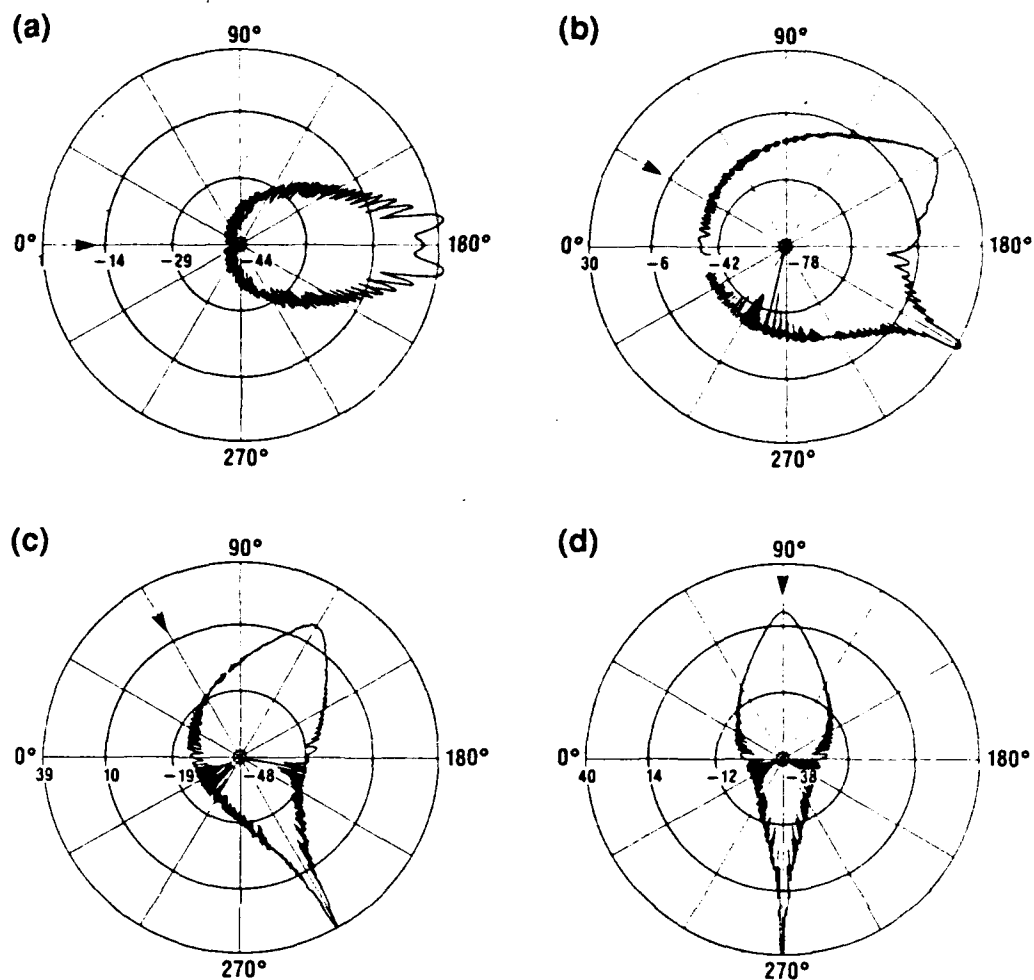


Fig. 3. Bistatic angular distribution for aspect ratio of 15 at  $kL/2=200$  (a) end-on, (b) 30 degrees relative to the axis of symmetry, (c) 60 degrees relative to the axis of symmetry, and (d) broadside.

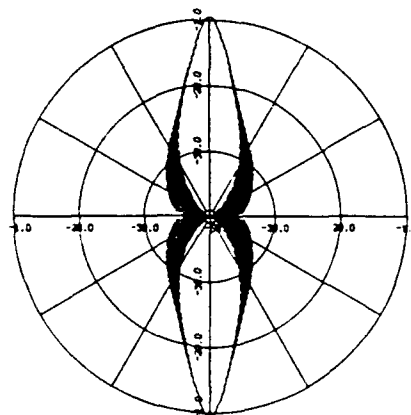


Fig. 4. Monostatic angular distribution for 15 to 1 spheroids for  $kL/2=120$ .

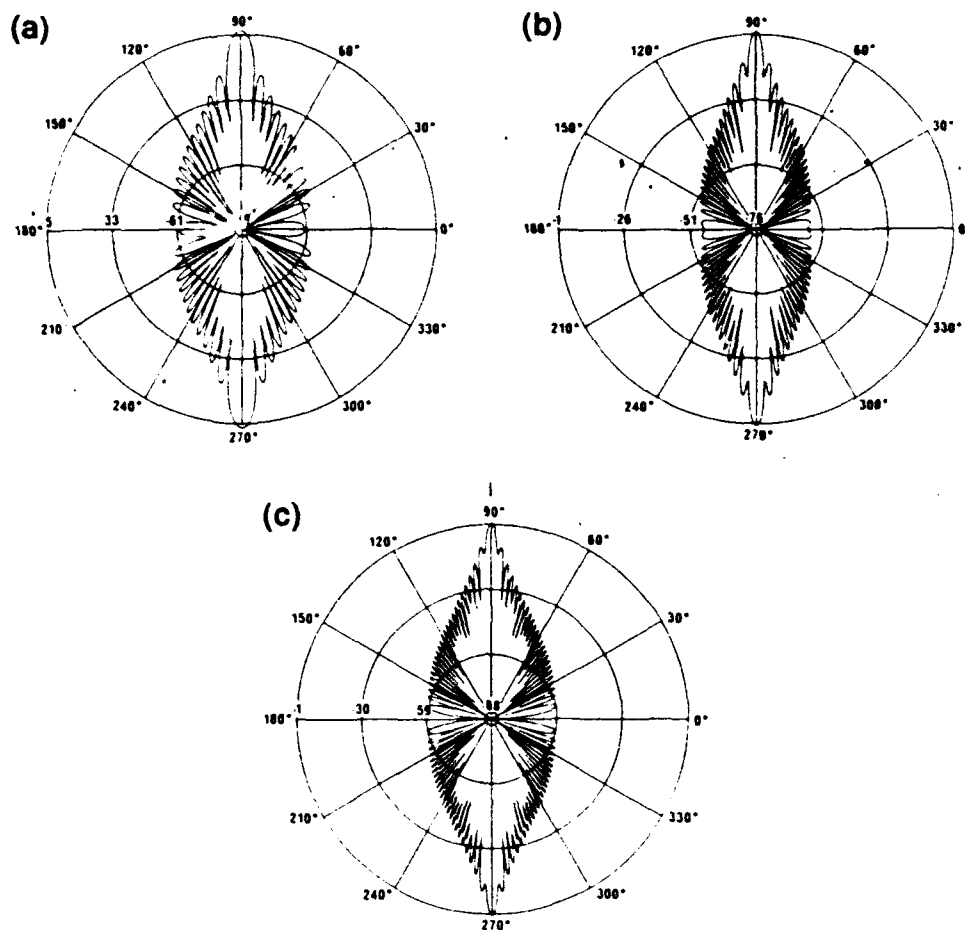


Fig. 5. Monostatic angular distribution for a 15 to 1 aspect ratio rigid spheroid at (a)  $kL/2=15$ , (b)  $kL/2=30$ , and (c)  $kL/2=60$ .